Development Aid and Multiple Steady-State Equilibria in a Neoclassical Growth Model

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Obstfeld (1999) has analysed the impact of development aid on the steady state and rate of convergence in a standard infinite-horizon neoclassical growth model. He found that when aid was disbursed as unconditional transfers to households, it had no effect on steady-state capital stock and output, but accelerated the pace of transition and raised steady-state consumption by the full amount of the aid inflow. We use the same model to analyse the outcome when aid is disbursed in the form of investment subsidies. We find that under this policy rule, the growth model has two saddle-path-stable steady states, one below and one above the no-aid benchmark. In an economy that is developing (in the sense of being below its steady state when aid commences), aid raises short-run consumption but the economy converges to a steady state below the no-aid case. In an economy that is initially above its no-aid steady state, aid causes convergence to a higher steady state. One implication is that conditional aid flows might cause divergence of capital stock and output per capita between poor and rich countries. A consequent implication for policy is that conditionality, in the sense of targeting aid to supplement investment, can have perverse effects on the long-run growth of poor countries.

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I. Introduction

There is an extensive empirical literature on the relationship between aid flows and country growth performance (Burnside and Dollar 2000, Collier and Dollar 2002, Easterly 2003, Guillaumont and Chauvet 2001, Hudson and Tarp 2001). Much of that work has regressed per capita income growth rates on aid flows and other explanatory variables, without specifying clearly the underlying theoretical model that is being tested. Implicitly or explicitly, much of this research has been based either on the Harrod-Domar framework used by Chenery and Strout (1966), or on some form of the closely-related AK model in which aid supplements domestic income and hence saving, causing a permanent acceleration in the rate of growth provided that the incremental capital-output ratio stays constant.[1]

Recently there has been renewed attention paid to the theoretical issue of how to incorporate aid into a neoclassical growth model.[2] Obstfeld (1999 pp.122-131) has added a permanent, exogenously-determined flow of aid into the representative household's budget constraint in an infinite-horizon intertemporal Ramsey-Koopmans-Cass model. He found that the steady-state capital-labour ratio and level of output were unaffected but that in the short run, in an economy which is initially below its steady-state capital stock, both consumption and investment would be increased by aid, speeding-up the pace of transition to the steady state. Thus, although the only long-run effect of aid in Obstfeld's model is to raise per-capita consumption by the full amount of the aid inflow, his model does predict a positive relation between aid inflow and the economy's rate of growth in the short run.

Dalgaard, Hansen and Tarp (2004) use an overlapping-generations model drawn from Diamond (1965) to explore the consequences of aid inflow in an infinite-horizon overlapping-generations model. In common with Obstfeld (1999) they analyse an economy which is closed except for aid inflows but, because the OLG model has two classes of agents (young and old), some policy rule is required to allocate aid between the two groups. A government is therefore introduced into their model to set the policy rule determining the allocation of aid transfers between old and young generations. The allocated aid transfers are then added into the budget constraints of the two groups, who proceed to consume or save their augmented budgets. Dalgaard et al show (2004 Appendix A) that provided the production function exhibits diminishing returns to capital, the model yields a unique steady state, which may be above or below the pure-autarky steady state level of capital and output per unit of labour according as the policy rule allocates aid preferentially towards the young or the old respectively. Intuitively this makes sense, since a policy rule which favours the old in aid allocation is in effect a subsidy to consumption because (i) the old do not save in this model, so all their allocation is consumed, while (ii) a reallocation of the aid flow towards the old reduces the incentive for the young to save and invest.

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[1] For a thorough review of the earlier literature on both capital flows and aid transfers see Eaton (1989), particularly sections 4.2 – 5.1 and section 7. For recent comments on the Burnside-Dollar work see Easterly 2003.

[2] Aid is here defined as international transfers, and is not to be confused with capital flows driven by differences in the marginal product of capital across countries, which present a separate set of theoretical challenges (Barro and Sala-i-Martin 2003 Chapter 3).

In this paper we address two issues which arise from the Obstfeld and Dalgaard et al papers. First, if a government decision-maker is to act as a "gatekeeper" responsible for the disbursement of aid inflows, as in Dalgaard et al (2004), some allocative policy rule and some channel for aid distribution must be specified. Unconditional transfers into household budgets - the distribution mechanism assumed by both Obstfeld (1999) and Dalgaard et al (2004) - is only one of several possible channels. Second, the possibility that aid inflow might open up multiple steady-state equilibria has not hitherto been seriously addressed. Though Dalgaard et al do draw attention to the formal possibility that constant or increasing returns to capital in the production function could yield more than one steady state equilibrium, they do not envisage any such effect from aid itself.

We use the same Ramsey-Koopmans-Cass intertemporal set-up as Obstfeld (1999), but we introduce a policy rule under which aid inflows are disbursed as investment subsidies rather than as unconditional transfers to households. (This seems plausible, given the common expectation among aid donors that aid should be targeted to promote investment.) We therefore assume that gross investment expenditure undertaken by domestic households attracts an aid-funded subsidy. Given this policy environment, we demonstrate the presence of two saddle-path stable steady states, and show that which of these two possible long-run equilibria the economy converges to will depend upon the level of development attained by the recipient economy prior to the commencement of aid inflows.

This result is of considerable interest, given the real-world evidence of divergence across countries over half a century of substantial international aid transfers after World War II (Pritchett 1996; Maddison 2001), and noting the existence of at least one significant group of economies (small islands) which seem to exhibit steady-state-like performance at very low levels of capital accumulation but relatively high per capita disposable incomes (Bertram and Watters 1985; Bertram 1986, 1999).

II. The Model

Consider an otherwise standard neoclassical Ramsey-Koopmans-Cass economy that consists of a continuum of agents with unit mass. The economy under consideration is closed to trade and private capital flows, but receives a constant inflow of international aid. The government distributes this aid in the form of an investment subsidy. The aid inflow takes the form of grants, so the economy's external indebtedness remains zero and there is no debt-servicing requirement. For simplicity, population remains constant and the rate of technical progress is assumed zero.

A. The Household's Problem

A representative household faces an instantaneous resource constraint

$$c + i = y = f(k),$$

where c is real consumption per capita, i is real gross private investment per capita, y is real output per capita, and k is real capital stock per capita. We assume $f'(k) > 0$ and $f''(k) < 0$, with Inada conditions being satisfied.

The capital transition equation for the household can be stated as

$$\dot{k} = (1+\sigma)i + q - \delta k \, ,$$

where σ denotes the rate of aid-funded investment subsidy per unit of gross investment, and q is a lump-sum investment subsidy.[3] A single representative household treats both σ and q as exogenous. The depreciation rate of capital is δ and the initial capital stock $k(0)$ is given.

Using the resource constraint to substitute out i in the transition equation yields

$$\dot{k} = (1+\sigma)[f(k) - c] + q - \delta k \, . \tag{1}$$

Let β stand for the pure rate of time preference. Then the representative household seeks to maximize its discounted lifetime utility by choosing the consumption trajectory which satisfies

$$Max. \int_0^\infty u(c) e^{-\beta t} dt$$

subject to condition (1). We shall assume log utility $u(c) = log(c)$.

If we let Λ represent the multiplier associated with (1), then the Hamiltonian for this problem can be formulated as

$$H = log\, ce^{-\beta t} + \Lambda \langle (1+\sigma)[f(k) - c] + q - \delta k \rangle \, .$$

The optimality conditions are

$$\frac{\partial H}{\partial c} = 0 : \qquad \frac{1}{c} e^{-\beta t} = \Lambda(1+\sigma); \tag{2}$$

$$\frac{\partial H}{\partial k} = -\dot{\Lambda} : \qquad \Lambda \langle (1+\sigma) f'(k) - \delta \rangle = -\dot{\Lambda} \, ; \tag{3}$$

$$\frac{\partial H}{\partial \Lambda} = \dot{k} : \qquad \dot{k} = (1+\sigma)[f(k) - c] + q - \delta k \, . \tag{4}$$

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[3] The assumption of a lump-sum component in the investment subsidy avoids having to interpret a steady state where the economy would accumulate no capital at all, which presents analytical difficulties if the Inada conditions are retained. For the purpose of focusing on a poor, developing economy, we think of q as a positive but rather small number.

To facilitate subsequent equilibrium analysis, it is helpful at this point to substitute out the co-state variable. Define

$$\Lambda \equiv \lambda e^{-\beta t} . \tag{5}$$

Then the time derivative of (5) is

$$\dot{\Lambda} = e^{-\beta t}\dot{\lambda} - \beta\lambda e^{-\beta t} . \tag{6}$$

Substituting (5) and (6) into (3) yields

$$-\dot{\lambda} = \lambda\langle(1+\sigma)f'(k) - \delta - \beta\rangle . \tag{7}$$

Next substitute (5) into (2):

$$\frac{1}{c} = (1+\sigma)\lambda \quad => \quad \lambda = \frac{1}{c}\cdot\frac{1}{1+\sigma} \tag{8}$$

The time derivative of (8) is

$$\dot{\lambda} = \frac{1}{1+\sigma}\cdot\frac{-1}{c^2}\dot{c} . \tag{9}$$

Substituting (8) and (9) into (7) gives

$$\frac{1}{1+\sigma}\cdot\frac{1}{c^2}\cdot\dot{c} = \frac{1}{c}\cdot\frac{1}{1+\sigma}\langle(1+\sigma)f'(k) - \delta - \beta\rangle . \tag{10}$$

$$=> \qquad \dot{c} = c\langle(1+\sigma)f'(k) - \delta - \beta\rangle . \tag{11}$$

Equations (4) and (11) completely characterize the household's intertemporal behavior. Having solved the household's problem, we now turn our attention to the government.

B. Government

The government in this economy receives a constant inflow of real resources (aid) from abroad, denoted by *x*, which it distributes in the form of an investment subsidy. The government budget constraint can be expressed as

$$\sigma i + q = x \quad \Rightarrow \quad \sigma[f(k) - c] + q = x . \tag{12}$$

Then so long as the government operates on its budget constraint, the subsidy rate σ is determined by the ratio of aid to household investment. Then equation (12) becomes

$$\sigma = \frac{x-q}{f(k)-c}. \tag{13}$$

C. Equilibrium Equations of Motion

Substitution of the government budget constraint (13) into the household's intertemporal equations (4) and (11) gives the minimum state-space representation of this economy:

$$\dot{k} = (1+\sigma)[f(k)-c]+q-\delta k$$

$$= \left[1+\frac{x-q}{f(k)-c}\right][f(k)-c]+q-\delta k$$

$$= \left[\frac{f(k)-c+x-q}{f(k)-c}\right][f(k)-c]+q-\delta k$$

$$= f(k)-c+x-\delta k \tag{14}$$

and

$$\dot{c} = c\left\langle\left[1+\frac{x-q}{f(k)-c}\right]f'(k)-\delta-\beta\right\rangle. \tag{15}$$

III. Steady State Analysis

For future reference, it is useful at this point to define k_R, such that

$$f'(k_R)-(\delta+\beta)=0. \tag{16}$$

Notice that definition (16) is the condition that defines the steady state capital stock in a conventional Ramsey growth model. The present model would revert to the standard Ramsey economy if there were no aid and therefore no subsidy scheme, i.e. $x=\sigma=q=0$. We shall refer to k_R as the Ramsey steady state.

The steady state(s) of the dynamic system in equations (1) – (15) can be characterized by imposing $\dot{k}=0$ and $\dot{c}=0$. For $c>0$, setting (14) and (15) to zero gives the steady state equations:

$$\dot{k}=0: \qquad c = f(k)+x-\delta k \tag{17}$$

and

$$\dot{c}=0: \qquad \left[1+\frac{x-q}{f(k)-c}\right]f'(k)=(\delta+\beta)$$

$$c=f(k)+\frac{(x-q)f'(k)}{f'(k)-(\delta+\beta)}. \tag{18}$$

Immediately it is clear that any steady state outcome in the present case cannot replicate the conventional Ramsey solution, given that (18) is undefined at k_R. Equating (17) and (18) yields an expression that characterizes the steady state capital stock:

$$g(k)\equiv\frac{(x-q)f'(k)}{f'(k)-(\delta+\beta)}+\delta k-x=0. \tag{19}$$

Equation (19) does not have any unique analytical solution. However, Appendix A shows that $g(k)=0$ exhibits two roots: k_L and k_H, such that $0<k_L<k_R<k_H$. This in turn establishes the existence of two steady states in this economy. Which steady state eventually prevails will depend on the initial capital stock. We defer this dynamic discussion until the next section.

One issue that immediately arises at this point is whether changing the amount of aid may shift the steady-state capital stock. Appendix B shows that if k_H is the relevant steady state, then an increase in aid will further increase the steady state capital stock. However, if k_L is the relevant steady state, the marginal effect of increased aid on the steady-state capital stock is ambiguous.

IV. Dynamic Analysis

In this section, we first construct the phase diagram for the minimum state-space representation in order to illustrate the dynamic properties of this economy. Then we conduct a policy experiment by using the phase diagram to show how our model economy's transition trajectory reacts to a sudden and permanent injection of foreign aid. This policy experiment also provides a comparison of results between our model, where aid specifically targets investment, and Obstfeld's (1999) model where aid is simply injected as an income supplement.

A. Phase Diagram Construction

Recall from the previous section that $\dot{k}=0$ implies (17). This locus intersects the consumption axis at *x*, i.e. $c=x$ when $k=0$. Furthermore, given the properties of the production function, namely $f'(k)>0$ and $f''(k)<0$, the locus initially increases but turns down when $f'(k)<\delta$. Figure 1 depicts the profile of this locus.

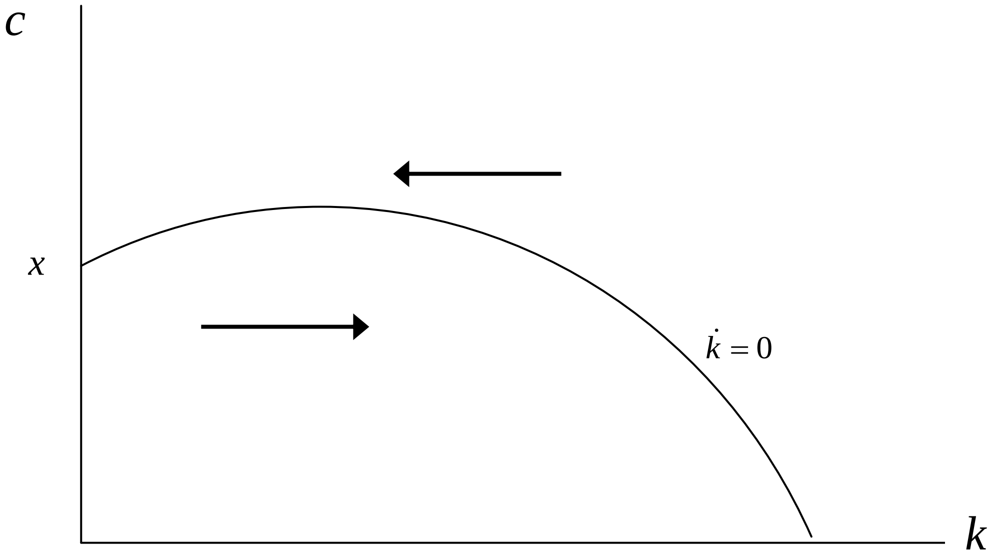

Fig. 1. – Steady-state locus for capital stock

At any point below the $\dot{k}=0$ locus, equation (14) shows that *k* is increasing because investment plus aid exceeds depreciation. Conversely, above the locus, *k* is falling.

Now consider the locus $\dot{c}=0$, which implies (18). Notice that (18) experiences discontinuity at k_R. Therefore $\dot{c}=0$ exhibits two arms on the phase diagram. One branch resides above and the other branch resides below k_R. In deriving the profile for $\dot{c}=0$, it is helpful to differentiate (18):

$$\frac{dc}{dk}=f'(k)-\frac{(\delta+\beta)(x-q)f''(k)}{\left[f'(k)-(\delta+\beta)\right]^2}>0\,. \tag{20}$$

and also to rewrite (18) slightly differently as

$$c=f(k)+\frac{x-q}{1-\frac{(\delta+\beta)}{f'(k)}}\,. \tag{21}$$

For $k<k_R$, according to (18), as *k* approaches k_R from below, *c* tends to positive infinity. Also observe (equation 21) that as *k* approaches zero, the Inada conditions mean that *c* approaches $x-q$. Furthermore the $\dot{c}=0$ locus is always increasing, as indicated by (20).[4] Combining these observations together generates the profile of $\dot{c}=0$ for $k<k_R$ as drawn in Figure 2.

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[4] Notice that according to (20), computing $\lim_{k\to 0}\frac{dc}{dk}$ would require us to place further assumptions on the third derivative of $f(k)$, which could tell us whether this branch of $\dot{c}=0$ had a very steep slope or not with *k* being close to zero. However, not imposing any third-derivative assumption does not alter

For $k > k_R$, according to (18), as k approaches k_R from above, c tends to negative infinity. Also, by inspecting (21) again and keeping in mind the Inada conditions, we observe that as k tends to positive infinity, c tends to positive infinity. Furthermore, condition (20) states that the $\dot{c} = 0$ locus is always increasing for this branch as well. Combining these observations together generates the profile of $\dot{c} = 0$ for $k > k_R$ as drawn in Figure 2.

To evaluate the dynamics in Figure 2, let $(\hat{k}, \hat{c})$ be any pair of values for which $\dot{c} = 0$. Then at any point $(\hat{k}, \overline{c})$ such that $\overline{c} > \hat{c}$, equation (15) shows that c will be increasing. Conversely, at a pair $(\hat{k}, \underline{c})$ such that $\underline{c} < \hat{c}$, c is falling. These directions are illustrated by the up-down arrows in Figure 2.

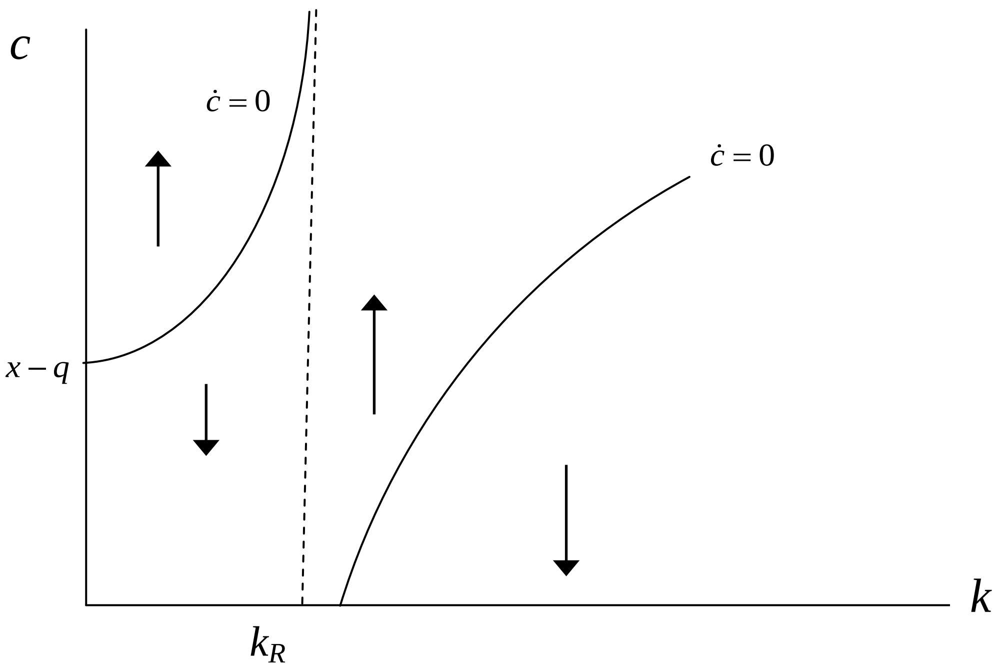

Fig. 2 – Steady state locus for consumption

Having analyzed $\dot{k} = 0$ and $\dot{c} = 0$ separately, the full phase diagram for our economic system can be constructed as Figure 3. The dynamic system exhibits two steady states, at A and B. Due to the perfect foresight nature of our model, for initial conditions that are close to the steady states, the phase diagram exhibits saddle-path stability associated with each of the two steady states.[5]

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significantly our qualitative discussion. Condition (20), namely $\frac{dc}{dk} > 0$, is sufficient for subsequent dynamic analysis.

[5] Given the non-linear nature of our system, analytical investigation of each steady state's local dynamic property is not possible. Appendix C provides a numerical example showing that the characteristic roots of the Jacobian matrix associated with each steady state are real, distinct, with mixed signs, which is in accordance with the saddle path dynamics depicted by the phase-diagram.

Given the existence of two steady states, which one applies to any particular economy will depend on whether the economy is initially above or below k_R. This has the intriguing implication that only when aid is given to an economy that is already relatively over-developed, compared to its Ramsey steady state at k_R, will the steady state at k_H be relevant. Any economy that is initially underdeveloped, when compared to k_R, will converge to k_L.

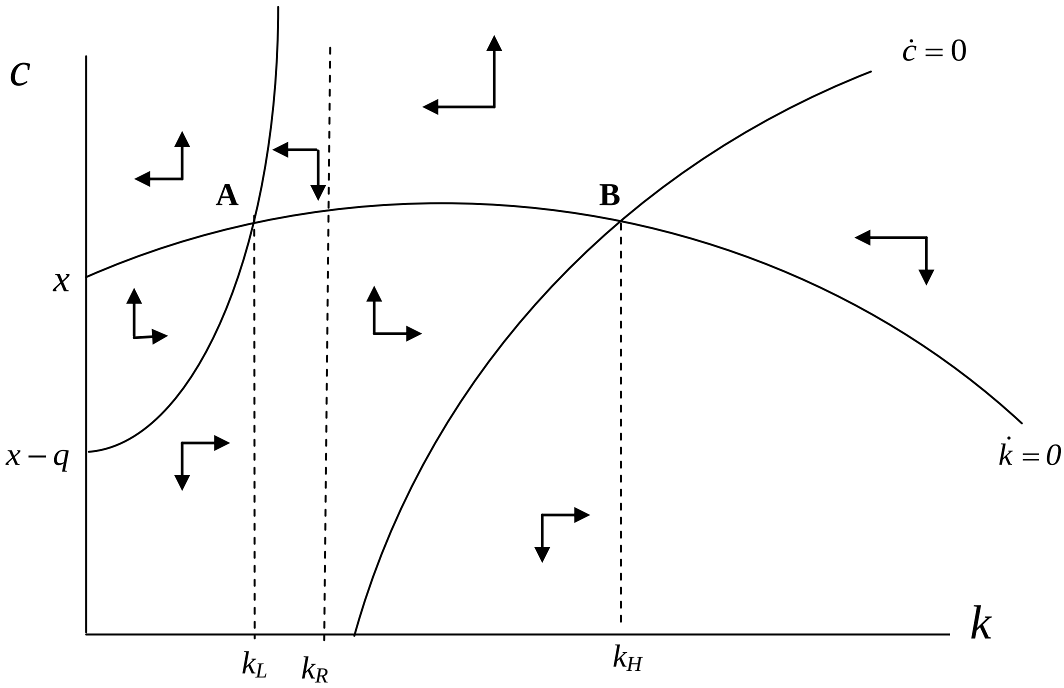

Fig. 3. – Two steady states

B. Policy Experiment

Consider a scenario where the economy initially does not receive any aid. As previously stated, if the economy receives no aid, and consequently there is no government subsidy to investment, the economy will converge to the standard Ramsey steady state. This situation is illustrated by the intersection of $\dot{k}=0_R$ and $\dot{c}=0_R$, point R in Figure 4.

Let $k(0)$ be the initial capital stock per capita. Then the economy without aid would be approaching its Ramsey steady state R along the saddle path $\mathrm{SP_R}$. Now consider an unexpected and permanent injection of aid into this economy, disbursed as an investment subsidy starting at $t=0$. After this policy shock, the dynamics are governed by $\dot{k}=0_A$, $\dot{c}=0_A$, and the new saddle path $\mathrm{SP_A}$. This implies that the economy shifts to its new saddle path by a jump in consumption, followed by a decrease in its capital stock and consumption per capita over time until A is reached. The arrows in Figure 4 depict the economy's movement over time.

This is in sharp contrast to Obstfeld's (1999) result. In Obstfeld's model where aid is injected simply as an income supplement to the household, c_R would have been the resulting steady state consumption, and the capital stock trajectory would not exhibit a decline if $k(0) < k_R$.[6] It is also obvious from the phase-diagram that c_R steady state consumption is superior to c_A, the steady state consumption level when aid is targeted to investment.

Therefore, the introduction of investment-targeted development aid to an originally isolated economy, that is initially in transition towards its Ramsey steady state capital stock k_R, causes the economy to switch to a growth path that converges on lower long-run per capita capital stock, output, and consumption. This result has the startling implication that in the present model, investment-targeted aid itself causes economic divergence and pushes the developing economy towards an inferior long-run equilibrium.

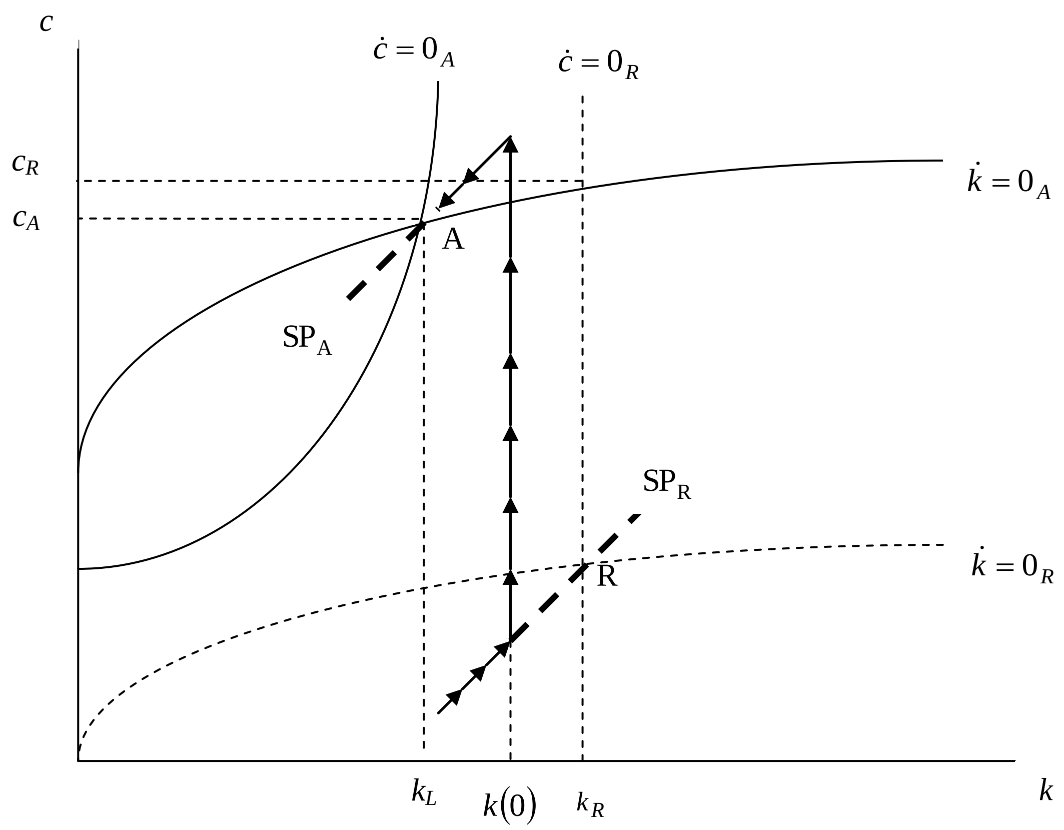

Fig. 4.

V. Conclusion

We have incorporated an external aid flow into an otherwise standard neoclassical growth model, together with a policy prescription that aid must be disbursed as an investment subsidy, rather than being allocated across households as unconditional

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[6] This can be easily verified by erasing the investment-target feature, i.e. setting σ and $\bar{z}$ to zero, but still retaining the aid, x. Then the dynamic system reduces to $\dot{k} = f(k) - c - \delta k + x$ and $\dot{c} = c[f'(k) - (\delta + \beta)]$.

transfers (the policy assumed in the recent papers of Obstfeld (1999) and Dalgaard el al (2004)). We find that this policy rule results in multiple steady state equilibria, in which a "more-developed" economy (initially above its Ramsey steady state) converges to a higher long-run capital stock and output than it would have reached without aid,[7] whereas a "developing" economy (one starting out below its Ramsey steady state) converges to a lower long-run equilibrium, with capital stock and output below what would have emerged without aid.

Obstfeld's (1999) analysis shows that aid which takes the form of unconditional transfers to households in the recipient country leaves the long-run output and capital stock unchanged at the Ramsey steady state level, but raises long-run consumption and speeds up the transition process, i.e. accelerates short-run growth. Our analysis shows that making aid conditional on investment performance has the effect, at least for developing countries, of reducing long-run capital stock and output relative to Obstfeld's case, and that this can entail a slow-down or reversal of capital accumulation.

One immediate policy implication of our result is that in a neoclassical growth environment, aid donors will secure a larger capital stock outcome in the long run for a poor recipient economy if the aid is distributed as unconditional transfers to households, than if the donors target their aid to direct investment subsidies. This further raises the possibility that the high costs to donors of monitoring and targeting aid flows and outcomes may be avoided by adoption of a simple policy rule that injects aid simply as income supplements for households.

A second important implication of this paper is that the recent econometric literature on aid effectiveness (reviewed by, e.g., Collier and Dollar 2004; Easterly 2003) has focused too narrowly on the interaction of aid flows with recipient-country policies and institutions, while largely ignoring the potentially important effect on growth of donor-government policies. Aid conditionality, such as the tying of aid to investment performance which we have assumed here, tends to be a policy stance of donors rather than of recipients.

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[7] This outcome resembles, to a certain extent, the effect of Marshall Plan aid to postwar West Germany, and possibly also US assistance to Japan at the same time. More recently, the response of some of the Gulf states (including Saudi Arabia) to the availability of large oil rents may reflect a similar process to that discussed in this paper.

Appendix A

Demonstrating that the dynamic system exhibits two steady states is equivalent to showing that equation (19), namely $g(k) \equiv \frac{(x-q)f'(k)}{f'(k)-(\delta+\beta)} + \delta k - x = 0$, has two roots.

First notice that except for $k = k_R$, where $g(k)$ is not defined, $g(k)$ is upward sloping:

$$\frac{dg}{dk} = -\frac{[f'(k)-(\delta+\beta)][(x-q)f''(k)] - [(x-q)f'(k)]f''(k)}{[f'(k)-(\delta+\beta)]^2} + \delta$$

$$= -\frac{(\delta+\beta)(x-q)f''(k)}{[f'(k)-(\delta+\beta)]^2} + \delta > 0. \qquad \text{(A1)}$$

Also note that, rewriting (19) as $g(k) \equiv \frac{x-q}{1-\frac{\delta+\beta}{f'(k)}} + \delta k - x = 0$, the Inada conditions imply that as $k \to 0, \quad f'(k) \to \infty \quad \Rightarrow \quad \frac{\delta+\beta}{f'(k)} \to 0$, and so $g(k) \to -q$

i.e. $$\lim_{k\to 0} g(k) = -q < 0 \qquad \text{(A2)}$$

and as $k \to k_R$ from below, $[f'(k)-(\delta+\beta)] \to 0 \quad \Rightarrow \quad g(k) \to \infty$

$$\lim_{k\to k_R^-} g(k) = \infty. \qquad \text{(A3)}$$

Combining (A1), (A2) and (A3), we can deduce that $g(k)=0$ has one root, k_L, such that $0 < k_L < k_R$.

Now observe that as $k \to k_R$ from above, $g(k) \to -\infty$, i.e.

$$\lim_{k\to k_R^+} g(k) = -\infty \qquad \text{(A4)}$$

and as $k \to \infty$, $f'(k) \to 0 \quad \Rightarrow \quad \frac{\delta+\beta}{f'(k)} \to \infty \quad \Rightarrow \quad g(k) \to \infty$, i.e.

$$\lim_{k\to\infty} g(k) = \infty \qquad \text{(A5)}$$

with the Inada conditions operating.

Combining (A1), (A4), and (A5), we can deduce that $g(k)=0$ exhibits another root, k_H, such that $k_R < k_H < \infty$.

Therefore the dynamic system has two steady states, k_L and k_H, such that $0 < k_L < k_R < k_H$.

Appendix B

Investigating the effect on the steady state capital stock given a marginal increase in aid requires the evaluation of $\frac{dk_L}{dx}$ and $\frac{dk_H}{dx}$. Rearranging (19), we know that in either steady state

$$(x-q)f'(k)+\delta k[f'(k)-(\delta+\beta)]-x[f'(k)-(\delta+\beta)]=0$$

$$=> \quad -qf'(k)+\delta kf'(k)-\delta(\delta+\beta)k+x(\delta+\beta)=0$$

Totally differentiating the above expression with respect to *k* and *x* yields

$$-qf''(k)dk+\delta[kf''(k)dk+f'(k)dk]-\delta(\delta+\beta)dk+(\delta+\beta)dx=0$$

$$=> \quad (\delta+\beta)dx=[qf''(k)-\delta kf''(k)-\delta f'(k)+\delta(\delta+\beta)]dk$$

Rearranging gives

$$\frac{dk}{dx}=\frac{\delta+\beta}{f''(k)(q-\delta k)-\delta[f'(k)-(\delta+\beta)]} \tag{B1}$$

Given that the steady states can not be solved analytically, direct evaluation of $\frac{dk_{steadystate}}{dx}$ is not possible. However, if the denominator of (B1) can be signed, then $\frac{dk}{dx}$ can be signed.

In either steady state with $\dot{k}=0$, equation (17) states that

$$c=[f(k)+x-\delta k]>0$$

$$=> \quad f(k)-c=i=(\delta k-x)>0$$

$$=> \quad (x-\delta k)<0$$

and since $x>q$, it must be the case that $(q-\delta k)<0$

Therefore, since $f''(k)<0$, we know that $[f''(k)(q-\delta k)]>0$.

The sign of (B1), therefore, will definitely be positive if $f'(k)<\delta+\beta$. At $k=k_H$, we know that this is the case. Hence $\frac{dk_H}{dx}>0$.

At the lower steady state where $k = k_L$, however, we know that $f'(k) > \delta + \beta$. Consequently, the sign of $\frac{dk_L}{dx}$ is ambiguous, and will be positive or negative according to the sign of $f''(k)(q - \delta k) - \delta[f'(k) - (\delta + \beta)]$.

Appendix C

Investigating local dynamics requires the computation of

$$J = \begin{bmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial c} \\ \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial k} \end{bmatrix}.$$

According to (14) and (15), the elements of *J* are

$$\frac{\partial \dot{k}}{\partial k} = f'(k) - \delta, \ \frac{\partial \dot{k}}{\partial c} = -1,$$

$$\frac{\partial \dot{c}}{\partial k} = c\left\langle \left[1 + \frac{x-q}{f(k)-c}\right] f''(k) + f'(k)\left[\frac{-(x-q)f'(k)}{(f(k)-c)^2}\right]\right\rangle$$
$$= c\left\langle \left[1 + \frac{x-q}{f(k)-c}\right] f''(k) - (x-q)\left[\frac{f'(k)}{f(k)-c}\right]^2 \right\rangle,$$

and

$$\frac{\partial \dot{c}}{\partial c} = c\left\langle \frac{(x-q)f'(k)}{[f(k)-c]^2} \right\rangle + \left\langle \left[1 + \frac{x-q}{f(k)-c}\right] f'(k) - (\delta + \beta) \right\rangle$$

With the additional assumptions: $f(k) = k^\alpha$, $\alpha = 0.5$, $\beta = 0.05$, $\delta = 0.1$, and $x = 1$, we can compute the steady states numerically.

Using (19), the high-capital-stock steady state is: $k_H = 22.144055$, with consumption $c_H = 3.4913415$. The characteristic roots of *J* associated with this steady state are $\gamma_1 = 0.23559727$ and $\gamma_2 = -0.1035744$.

At the low-capital-stock steady state: $k_L = 1.7343646$, with consumption $c_L = 2.1435163$. The characteristic roots of *J* associated with this steady state are $\theta_1 = 1.0239416$ and $\theta_2 = -0.1486918$.

Given that for either steady state the characteristic roots are real, distinct, with mixed signs, the computed steady states therefore are locally saddle-path stable.

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